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Comparison of hysteric and adhesive coefficient of friction for rubbers sliding onto self-affine rough surfaces

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In this paper we investigate the influence of both the hysteric and adhesive friction for rubber surfaces sliding on self-affine rough surfaces. This type of roughness is described by the roughness amplitude w , the in-plane correlation length ξ , and the roughness exponent H that characterizes the degree of surface irregularity at short length scales ($< \xi$). It is shown that beyond the typical roughness parameters w and ξ , which are usually used in describing rough surfaces, the influence of the roughness exponent H should be taken carefully into account in order to properly gauge the contributing frictional mechanisms. Indeed, it is shown that for large roughness exponents ($H \sim 1$) and/or small roughness ratios $w/\xi < 10^{-2}$, the adhesive friction becomes significant at relatively high sliding velocities beyond the maximum of the hysteric component. © 2005 American Institute of Physics. [DOI: 10.1063/1.1844617]

I. INTRODUCTION

The friction which develops between a rubber body sliding onto a hard solid surface is important from the fundamental and technological point of view in car industry (tire construction, and wiper rubber blades), cosmetic industry, etc.¹⁻⁴ The major difference in the frictional properties of rubbers with respect to other solids arises from their low elastic modulus E , and the high internal friction that is present over a wide frequency range.⁵ In any case, the friction force between a rubber body and a hard rough solid substrate has two major contributions, which are the hysteric, and the adhesive ones.¹

The hysteric component arises from the oscillating forces that the surface asperities exert onto the rubber surface, leading effectively to cyclic deformations and energy dissipation due to internal frictional damping.⁵ As a result, the hysteric contribution will have the same temperature dependence as that of an elastic complex modulus $E(\omega)$.⁵ On the other hand, the adhesive component is important for clean and relatively smoother surfaces.⁵ In addition, depending on the sliding velocity, the low elastic modulus of rubbers leads to instabilities at high sliding velocities and for relatively smooth surfaces (Schallamach waves¹). In this case, a compressed rubber surface in front of the contact area undergoes a buckling producing detachment waves from the front-end to the back-end of the contact area. This case is excluded since we consider low sliding speeds.⁵

For rubbers and other elastically soft solids a weak adhesive junction due to van der Waals interactions between the surfaces may be well elongated before it breaks at a distance that is larger than the size of the surface asperities.⁶ Thus, during the block sliding a large fraction of the junctions will be simultaneously (elastically) elongated and exert a force on the moving body in contact with the rough sub-

strate. Furthermore, sliding onto real solid surfaces occurs in many cases onto rough surfaces with a significant degree of randomness.⁷⁻¹⁰ The latter implies that these surfaces possess roughness over various length scales rather than a single one. This is a fact, which has to be taken carefully into account in contact-related phenomena (i.e., friction and adhesion).^{5,6}

Up to now, it has been shown that for self-affine random rough surfaces, the coefficient of hysteric friction depends significantly on the roughness exponent H ($0 \leq H \leq 1$), which characterizes the degree of surface irregularity at short length scales.^{5,7} Nevertheless, the previous studies did not investigate how self-affine roughness influences both the hysteric and adhesive components of friction, which will be considered in this article by the inclusion of contributions from all lateral roughness wavelengths without relying on power-law approximations for the self-affine roughness spectrum,⁵ which is valid for lateral roughness wavelengths of $q\xi > 1$ with ξ as the in-plane roughness correlation length.

II. CONCISE THEORY OF FRICTION

A. Hysteric component

If a rubber body slides with velocity V over a sinusoidal rough surface with period L , then it will feel fluctuating forces with a characteristic frequency $\sim V/L$. In addition, if the surface has a wider distribution of length scales L , then it will present a wider distribution of frequency components in the Fourier decomposition of the surface stresses acting on the sliding rubber.⁵ For a rubber body of Young modulus E and Poisson ratio ν that slides onto a solid rough surface with velocity V , the coefficient of hysteric friction is given by⁵

$$\mu_{\text{HF}} = \frac{1}{2} \int_{q_{\text{con}}}^{Q_c} q^3 C(q) P(q) dq \times \int_0^{2\pi} \text{Im} \left[\frac{E^* (qV\tau \cos \phi)}{(1 - \nu^2)\sigma} \right] \cos \phi d\phi, \quad (1)$$

where $Q_c = \pi/a_o$ with a_o of microscopic dimensions, and

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$q_{\text{con}} = 2\pi/\lambda$ with λ is of the order of the diameter of the nominal contact area. The contact factor $P(q)$ is given by⁵

$$P(q) = \frac{2}{\pi} \int_0^{+\infty} \frac{\sin x}{x} e^{-x^2 G(q)} dx, \quad (2)$$

$$G(q) = \frac{1}{8} \int_{q_{\text{con}}}^q q^3 C(q) dq \int_0^{2\pi} \left| \frac{E(qV\tau \cos \phi)}{(1-\nu^2)\sigma} \right|^2 d\phi$$

and represents the fraction of the original nominal contact area, where contact remains when we study the contact area on the length scale $2\pi/q$.⁵ $C(q)$ is the Fourier transform of the correlation function $C(r) = \langle h(r)h(0) \rangle$ with $h(r)$ the surface roughness height ($\langle h \rangle = 0$). $\langle \cdots \rangle$ is an ensemble average over possible roughness configurations. σ is the applied macroscopic load, and $E^*(\omega)$ is the complex conjugate $E(\omega)$.

B. Adhesive component

If we assume uniaxial deformations of a cylindrical bar, the main energy dissipation within a viscoelastic medium covering a surface asperity of volume $V_b (\approx L^3)$ is given by⁶

$$\Delta E = \frac{V_b}{2\pi} \int \omega \text{Im} \left[\frac{1}{E(\omega)} \right] |\tilde{\sigma}(\omega)|^2 d\omega. \quad (3)$$

Denoting by $\tilde{\sigma} = \sigma_o \cos(\omega_o t)$ the fluctuating stress ($\sigma_o = F_N/L^2$ and F_N the normal force) and setting $\Delta E = VTF_{\text{AF}}$, where F_{AF} is the frictional force due to adhesion and T an oscillation period, we obtain for the adhesive friction coefficient of $\mu_{\text{AF}} = F_{\text{AF}}/F_N = \sigma_o \text{Im}[1/E(\omega_o)]$.⁶ In order to take into account the dependence of the stress factor σ_o on contact details over a distribution of lateral length scales that is present for random rough surfaces, we proceed as follows. If U_{el} is the energy needed to push the rubber body into contact with the rough substrate over macroscopic dimensions, and $w = \sqrt{\langle h^2 \rangle}$ is the rms roughness amplitude which represents the magnitude of effective depth that the rubber will have to be pressed in order to stay in contact with the rough substrate surface. We can calculate the average normal stress by the relation,^{11,12}

$$\langle \sigma_o \rangle = \frac{1}{A_{\text{flat}}} \left(\frac{\partial U_{\text{el}}}{\partial w} \right)$$

with $U_{\text{el}} = A_{\text{flat}} \frac{|E(\omega_o)|}{4(1-\nu^2)} \int_{Q_\lambda}^{Q_c} q C(q) d^2 q.$ (4)

Therefore, the adhesive friction coefficient is given in this case by^{11,12}

$$\mu_{\text{AF}} = \frac{|E(\omega_o)|}{4(1-\nu^2)} \left\{ \int_{q_{\text{con}}}^{Q_c} q \frac{\partial C(q)}{\partial w} d^2 q \right\} \text{Im} \left[\frac{1}{E(\omega_o)} \right], \quad (5)$$

where A_{flat} is the average macroscopic flat surface area. The characteristic frequency ω_o is given by $\omega_o = V/\xi$, and we ignore any weak frequency dependence of the Poisson ratio ν .^{5,6}

III. RESULTS AND DISCUSSION

From Eqs. (1) and (4) we have for the total coefficient of friction $\mu_{\text{total}} = \mu_{\text{AF}} + \mu_{\text{HF}}$ where any tearing processes are ignored. For the modulus $E(\omega)$ we consider the model⁵

$$E(\omega) = \frac{E(\infty)[(1+\alpha) + (\omega\tau)^2]}{(1+\alpha)^2 + (\omega\tau)^2} - j \frac{\alpha\omega\tau E(\infty)}{(1+\alpha)^2 + (\omega\tau)^2} \quad (6)$$

with $E(\infty)/E(0) = 1 + \alpha$ (typically $\alpha = 10^3$).⁵ $1/\tau$ is the flip rate of molecular segments that are configuration changes responsible for the rubber viscoelastic properties.

Furthermore, as Eqs. (1) and (5) indicate, in order to calculate the total coefficient of friction μ_{total} the knowledge of the spectrum $C(q)$ is necessary. A wide variety of surfaces/interfaces is well described by a kind of roughness associated with self-affine fractal scaling,⁷ for which $C(q)$ scales as a power-law $C(q) \propto q^{-2-2H}$ if $q\xi \gg 1$, and $C(q) \propto \text{const}$ if $q\xi \ll 1$.⁷ The roughness exponent H is a measure of the degree of surface irregularity,⁷ such that small values of H characterize more jagged or irregular surfaces at short length scales ($< \xi$). The self-affine scaling behavior is satisfied by the simple model⁸

$$C(q) = \frac{1}{2\pi} \frac{w^2 \xi^2}{(1 + a q^2 \xi^2)^{1+H}} \quad (7)$$

with $a = (1/2H)[1 - (1 + a Q_c^2 \xi^2)^{-H}]$ if $0 < H < 1$ (power-law roughness).⁸ For other models, see Refs. 9 and 10.

Our calculations were performed for $a_o = 0.3$ nm, $\nu = 0.5$, ratio $E(\infty)/\sigma = 1$ in Eq. (1), $E(\infty) = 10^4$ Pa, and $1/\tau = 5 \times 10^3$ s⁻¹. Moreover, since $C(q) \propto w^2$, for the adhesive friction coefficient we have $\mu_{\text{AF}} \sim w$, while for the hysteric component the situation is more complex because of the factor $P(q)$. Therefore, any complex dependence of the coefficient of friction μ_{total} on the substrate roughness will arise from all the roughness parameters w , H , and ξ . As shown in Fig. 1, with increasing rms roughness amplitude w the total coefficient of friction $\mu_{\text{total}} = \mu_{\text{AF}} + \mu_{\text{HF}}$ follows the behavior of the hysteric component μ_{HF} at higher sliding velocities. In both cases the effect of the adhesive friction becomes significant at relatively high sliding velocities so that the viscoelastic system is above the maximum of the hysteric component (Fig. 1). Around the maximum the dominant role is solely played by the hysteric component.

Besides the roughness amplitude, the friction coefficient μ_{total} as function of the lateral correlation length ξ shows similar behavior, as Fig. 2 indicates. Indeed, surface smoothing by an increment of the correlation length leads to a comparable effect in magnitude with that of the roughness amplitude w . The roughness parameters w and ξ characterize the long length scale ($> \xi$) behavior of the roughness features. In addition, roughening at short length scales as characterized by the roughness exponent H has a significant effect on the adhesive friction. Indeed, with increasing the roughness exponent H the influence of the adhesive friction becomes evident at lower sliding velocities, as Fig. 3 shows. In both Figs. 2 and 3 the change of the correlation length ξ and/or the roughness exponent H within a reasonable range of values (not more than an order of magnitude) leads to significant changes in the total coefficient of friction. This is because the

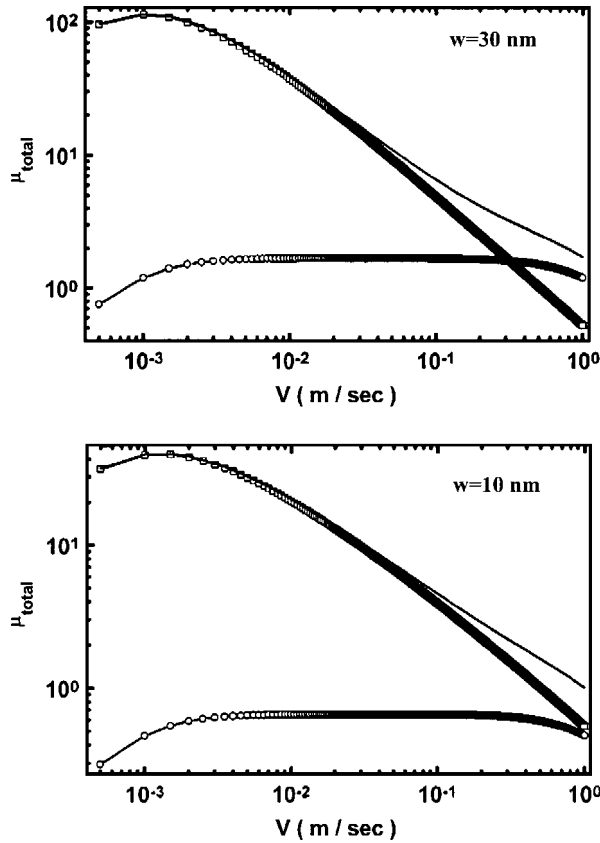


FIG. 1. Total friction coefficient μ_{total} (solid line) vs sliding velocity V for $H=0.5$, $\xi=200$ nm and various roughness amplitudes w as indicated. For comparison, the squares show the hysteric coefficient μ_{HF} , and the circles the adhesive coefficient μ_{AF} .

local surface slope $\rho = \sqrt{\langle |\nabla h|^2 \rangle}$ is very sensitive to changes of the roughness exponent H in the range of $0 < H < 1$ and the correlation length since $\rho \propto w/\xi^H$.¹¹

Our results show that the influence of adhesive friction is significant for large roughness exponents $H \sim 1$, and/or long-wavelength roughness ratio $w/\xi \leq 10^{-2}$. Some analytic results are possible in this case of weak roughness where the influence of adhesion is distinguishable. Indeed, for the adhesive friction we have for the roughness exponent $H=1$ the simple analytic expression,

$$\mu_{\text{AF}}|_{H=1} = \frac{2w}{w_o^2} \left\{ \frac{1}{a^{3/2}\xi} [\tan^{-1}(\sqrt{a}\xi Q_c)] - \frac{1}{2a} \left(\frac{Q_c}{1 + a\xi^2 Q_c^2} \right) \right\} \quad (8)$$

with $w_o = \sqrt{4(1-\nu^2)/|E(\omega_o)|}$. It should be pointed out that the influence of the adhesive friction is distinguishable when the decreasing hysteric component intersects the rather plateau area of the adhesive component where it is significant in magnitude. Figure 4 shows the range of velocities after which the ratio $\mu_{\text{AF}}/\mu_{\text{HF}}$ becomes larger than unity. The influence of the roughness exponent H appears clearly distinct, where for large roughness exponents or smoother surfaces at short length scales ($< \xi$) the influence of adhesion is more prominent. Similar is the influence of the roughness ratio w/ξ upon decrement or long-wavelength surface smoothing.

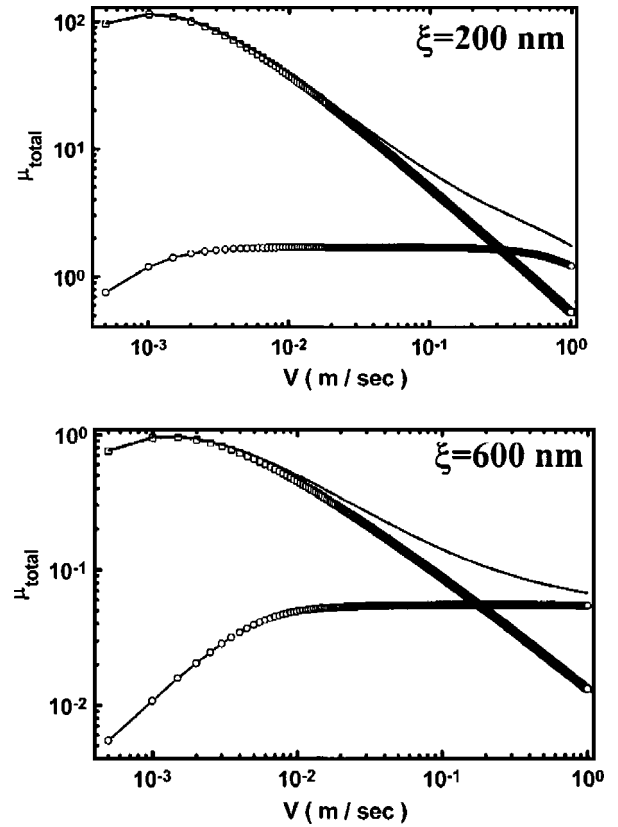


FIG. 2. Total friction coefficient μ_{total} (solid line) vs sliding velocity V for $w=10$ nm, $H=0.5$, and various correlation lengths ξ as indicated. For comparison, the squares show the hysteric coefficient μ_{HF} , and the circles the adhesive coefficient μ_{AF} .

We should point out, however, that our previous discussion is valid as long as instabilities at high sliding velocities (Schallamach waves¹) do not take place.

Finally, we should point out that in actual situations besides adhesive and hysteric friction, the rubber produces traction forces through tear and wear. As deformation stresses and sliding speeds increase (e.g., tires in racing cars), the local stress can exceed the tensile strength of the rubber especially near the point of a sharp irregularity. The high local stress can deform the internal rubber structure beyond the point of elastic recovery. Indeed, when polymer bonds and cross-links are stressed to failure the material can no longer recover completely, leading to tearing. The latter absorbs energy and results in additional friction forces within the contact surface. The wear processes are the ultimate result of tearing.

IV. CONCLUSIONS

In summary, we investigated the influence of hysteric and adhesive friction for rubber surfaces sliding on self-affine rough surfaces. It is shown that for large roughness exponents ($H \sim 1$) and/or small roughness ratios $w/\xi < 10^{-2}$ the adhesive friction becomes significant at higher sliding velocities beyond the maximum of the hysteric component and closer to the glassy regime for the rubber. Besides the typical roughness parameters w and ξ , which are used in describing rough surfaces, the present study shows that the

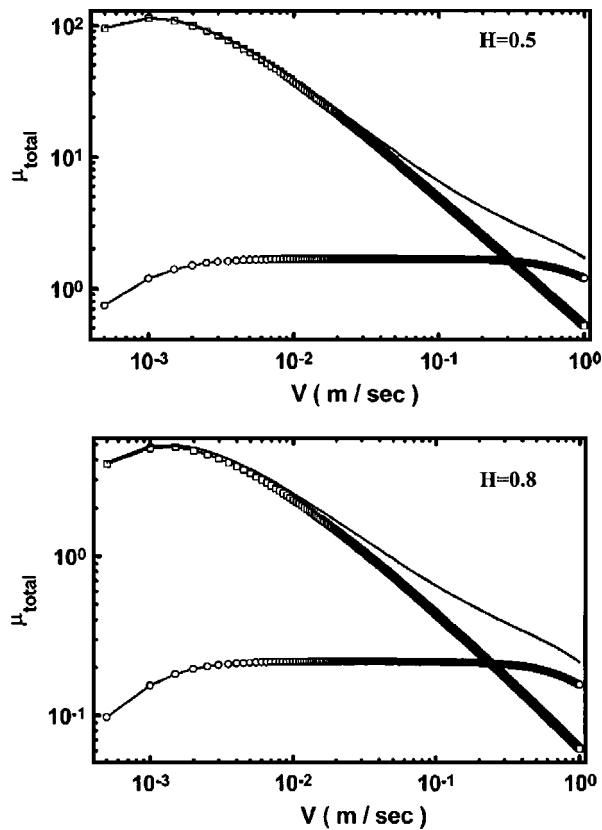


FIG. 3. Total friction coefficient (solid line) μ_{total} vs sliding velocity V for $w=10$ nm, $\xi=200$ nm, and various roughness exponents H as indicated. For comparison, the squares show the hysteric coefficient μ_{HF} , and the circles the adhesive coefficient μ_{AF} .

influence of the roughness exponent H should be taken carefully into account in friction-related phenomena in order to gauge properly the contributing mechanisms at various sliding velocities.

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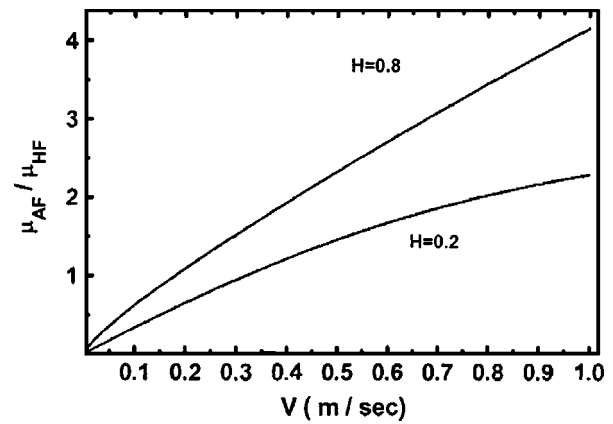


FIG. 4. Ratio $\mu_{\text{AF}}/\mu_{\text{HF}}$ vs sliding velocity V for $w=10$ nm, $\xi=200$ nm, and various roughness exponents H as indicated.

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